DOCTORAL (PHD) DISSERTATION SUMMARY

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< The Effect of Teaching Conceptual Knowledge on Students' Achievement in, Anxiety About, and Attitude Toward Mathematics in the Kurdistan Region of Iraq>

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SUMMARY

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<The Effect of Teaching Conceptual Knowledge on Students' Achievement in, Anxiety About, and Attitude Toward Mathematics in the Kurdistan Region of Iraq>

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Abstract

While thus far mathematics researchers have tended to concentrate on procedural knowledge, in the last few decades, there has been increasing interest in conceptual knowledge. Therefore, the present dissertation highlights the importance of teaching mathematics conceptually alongside the teaching of procedural knowledge from researchers' and educators' perspectives. In addition, it investigates how teaching for conceptual understanding affects students' achievement in, anxiety about, and attitude toward mathematics.

This study draws on interviews with thirty secondary school mathematics teachers from the Erbil city in the Kurdistan Region of Iraq, regarding their views on the conceptual aspect of mathematical knowledge. The three main aspects of the study are focused on: mathematics teacher's perspectives on teaching mathematics conceptually; mathematics teachers' need to teach conceptually, and the obstacles that face them in teaching mathematics conceptually. Furthermore, an experimental approach is utilized to evaluate 200 secondary school students from the same area. In the experimental group, conceptual teaching was the focus. While, in the control group, conventional teaching was used. Pretests and posttests for an achievement test, abbreviated Math Anxiety Scale, and Mathematics Attitude Scale were applied to both the treatment and control groups to reveal the effect of conceptual knowledge on students' achievement in, anxiety about, and attitude toward mathematics, respectively.

A thematic analysis of the interviews with secondary school mathematics teachers reveals that they believe that conceptual knowledge is as important as procedural knowledge. They believe that achieving a balance between conceptual and procedural understanding as well as connections between them, are necessary for understanding real mathematics. Furthermore, the pretest and posttest results with secondary school students show that there is a statistically significant difference in mathematics achievement between the two groups ($p < .001$). Students' attitudes toward mathematics in the treatment group developed positively. Nevertheless, teaching mathematics conceptually reduced anxiety among female students more effective than it did among male ones.

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1. Introduction

1.1. The Significance of The Study

Mathematics researchers, in the last few decades, have concentrated on the necessity of conceptual teaching for students' success in learning mathematics (see, for example, Crooks & Alibali, 2014). According to the National Council of Teachers of Mathematics (2000), for students to be successful in learning mathematics, they have to learn with understanding. The reason for students' low performance in Calculus was the lack of conceptual understanding (Liang & Martin, 2008).

Educators and researchers must consider anxiety about and attitude toward mathematics because these are serious problems among students (Christiansen, 2021; Webb, 2017). The concept of attitude is used and understood in the way Pehkonen and Pietilä (2003) defined as a psychological construct that belongs to the affective-emotional side of human personality. Anxiety also belongs to the emotional sphere of personality, but definitely to the negative side (Hembree, 1990).

To sum up, students understanding of mathematics and their confidence regarding their ability can be developed by providing them interactive teaching methods in a relaxing environment that leads to improve their attitudes toward mathematics (Jennison & Beswick, 2010). Therefore, mathematics educators must focus on using a holistic teaching approach to interact the students with mathematics subjects and to improve their positive attitude toward mathematics (Turner et al., 2002).

1.2. Problem Statement

Numerous mathematics educators tend to use conventional teaching methods that concentrate on procedural teaching and neglect the important aspect of teaching which is conceptual teaching. Mathematics teachers define mathematics as provider a set of tools, such as problem-solving skills, logical reasoning, and thinking ability abstractly (Andamon & Tan, 2018). Based on their definition mathematics teachers depend on procedural teaching rather than conceptual teaching which is an insufficient approach to improving the students' mathematics competence. Students understanding mathematics conceptually, however, helps them to develop their confidence and decline their mathematics anxiety. This leads to an increase in their ability in confronting mathematics challenging tasks more easily and trustfully (Mariquit & Luna, 2017).

According to a study conducted by Zaini (2005) on the teaching of conceptual knowledge, the result revealed that trainee teachers depended on algorithms, rules, and formulas to explain problems instead of evidence-based understanding. Likewise, in Saudi Arabia primary school mathematics teachers depended more on procedural knowledge than conceptual knowledge (Khashan et al., 2014). Consequently, students learn only rules and depend on them to confront in solving mathematics problems which is insufficient for solving the problems that require deep understanding (i.e., non-traditional problems). For instance, in Hussein and Csíkos (2021) study revealed that secondary school students in the Kurdistan region of Iraq had many problems with the concept of function. The study showed that students had difficulty defining the concept of function, and they tended to conflate concept image with concept definition. They could not provide a complete and clear definition of function but were only able to provide the definition partially. In addition, they had difficulty recognizing different representations of functions and conversions between different modes of representation.

Teaching mathematics procedurally, which is the common teaching method, increases concerns about students' performance and anxiety about mathematics. Khoule et al. (2017) found in their study that teaching procedurally not only does not overcome students' mathematics anxiety, but it helps to arise their mathematics anxiety. Because this method of teaching focuses on mastering rules without understanding and students should remember them in the exam. This stress of remembering the materials that students studied, in the exam, increases their anxiety (Khoule et al., 2017).

1.3. Hypothesis

The hypotheses are formed as null hypotheses. This was done for the purpose of straightforward testing them, and based on the literature the researcher's real expectations can be formed as the alternative hypotheses of the following.

- In terms of students' achievement, there will be no statistically significant difference between the control group and the experimental group.
- In terms of decreasing students' anxiety, there will be no statistically significant difference between the control group and the experimental group.
- In terms of improving students' positive attitudes toward mathematics, there will be no statistically significant difference between the control group and the experimental group.

1.4. Purpose of The Study and Research Questions

The purpose of this dissertation is to examine mathematics teachers' perspectives on the necessity of teaching mathematics conceptually and the obstacles that they face in this endeavor. In addition, it investigates how secondary school students' conceptual knowledge impacts their achievement in, anxiety about, and attitude toward mathematics in the Kurdistan region of Iraq. It aims to disseminate results and contribute to improving the teaching of mathematics.

The study is guided by the following research questions:

1. What is the importance of conceptual knowledge in teaching mathematics for students from mathematics teachers' perspectives?

   This question has four sub-questions:
   a. What is mathematics teachers' familiarity with conceptual understanding?
   b. What are mathematics teachers' perspectives on teaching mathematics conceptually?
   c. What do mathematics teachers need to teach conceptually?
   d. What are the obstacles that mathematics teachers face when teaching mathematics conceptually?
2. Does teaching mathematics conceptually affect students' achievement?
3. Does teaching mathematics conceptually affect students' anxiety?
4. Does teaching mathematics conceptually affect students' attitudes?

2. Literature Review

2.1. Importance of Conceptual Knowledge in Teaching Mathematics

Investigating conceptual knowledge helps learners gain procedural knowledge. In Lauritzen's study (2012), students who scored highly on conceptual tasks also scored highly on procedural tasks. However, a low level of conceptual knowledge was recorded among first-year students in the mathematics department at the College of Education (Saeed, 2016). When students are asked to solve a mathematical problem, they can use processes to find the correct solution despite lacking an understanding of "how" and "why" (Barr et al., 2003). Therefore, "the results support the genetic view that procedural knowledge is a necessary but not sufficient condition for conceptual knowledge" (Lauritzen, 2012, p. 13)

Many studies have indicated that a lack of conceptual knowledge leads to a variety of challenges. For example, students have difficulty with algebraic concepts such as algebraic expressions due to a lack of conceptual knowledge (Tekin-Sitrava, 2017). In Rittle-Johnson and Alibali's (1999) study, equivalent tasks were provided to students, and they were asked to decide which one was correct and which one had no meaning. The study found that 86% of participants failed to solve the problems because they lacked basic arithmetic skills. In addition, a study by Carlson (1998) found that university students were unable to solve an unconventional problem in the development of the concept of a function. Specific problems have been identified in the research. For example, in calculus, derivation was found to be particularly difficult for most undergraduate students to understand (Saha et al., 2010). According to Willingham (2009), students did not understand the base of ten numbers system totally, and only twenty-five percent of sixth-grade students have a deep understanding of the equal sign. These difficulties are believed to result from students' lack of conceptual understanding of the concepts (Saha et al., 2010; Willcox & Bounova, 2004). Therefore, a lack of conceptual knowledge is a reason for students' weak performance in mathematics (Knuth et al., 2005).

2.2.Relationship Between Conceptual and Procedural Knowledge

Mathematics educators believe that both conceptual and procedural knowledge are essential (Hiebert & Grouws, 2007; Hurrell, 2021; Rittle-Johnson et al., 2015). For instance, Kilpatrick et al. (2001) stated that procedural knowledge is primarily needed to support conceptual knowledge. Procedural knowledge is an important aspect of mathematical proficiency, it is also important to develop the ability to understand the underlying principles and relationships that drive mathematical concepts and procedures. The basis for mathematical fluency and problem-solving skill is a combination of procedural knowledge and conceptual comprehension. Therefore, conceptual knowledge and procedural knowledge are both necessary and help to strengthen each other (Hurrell, 2021), and connecting these two types of knowledge is the key to developing mathematical understanding (Aydın, 2018; Baroody et al., 2007; Hiebert & Lefevre, 1986). Similarly, simultaneously developing these two types of knowledge has a positive effect on mathematical competence (Rittle-Johnson et al., 2015). Accordingly, conceptual understanding is supported by algorithms and provides building blocks that can be used to clarify concepts. Conversely, students can develop algorithms through conceptual understanding (Aydın, 2014). Therefore, conceptual and procedural knowledge are often mentioned together because it is believed that they have a coherent relationship between them (Rittle-Johnson & Schneider, 2015).

2.3.Mathematics Anxiety

Mathematics anxiety has been a central issue in education for many decades (Khoule, et al., 2017). Dreger and Aiken (1957) first suggested that there is a specific type of anxiety called numerical anxiety. Fifty years ago, Richardson and Suinn (1972) produce the first instrument to measure mathematical anxiety. Their questionnaire was developed over the following decades by shortening the original 98-item questionnaire (see, for example, Hopko et. al, 2003). One-third of those who consult a university student counselor have mathematics anxiety (Richardson & Suinn, 1972). More than 30% of 15 year-old students in OECD countries reported mathematics anxiety in the 2012 PISA survey (OECD, 2013). A study conducted by Curtain-Phillips (1999) on mathematics anxiety found that mathematics anxiety was common among students. According to Rossnan (2006), children struggle with mathematics anxiety that hinders their ability to understand mathematics as a part of their daily lives. Also, a study conducted by Jackson and Leffingwell (1999) on first-year college students, revealed that a huge amount of the participants had mathematics anxiety. Furthermore, it has the main role in students choosing their future careers. Majors that include high mathematics requirements are avoided by university students who have mathematics anxiety (Lefevre et al., 1992; Widmer & Chavez, 1982). Many people's fears of mathematics lead to hinder them from pursuing specific professional opportunities (Tobias, 1993). Therefore, it is vital to reduce students' mathematics anxiety because it is a barrier to improving their mathematics performance (Capinding, 2022).

2.4.Attitude Toward Mathematics

Students' different experiences with mathematics form their attitudes toward it. They develop a positive or negative attitude because their accumulated experiences with a subject affect their psychological state (Sunghwan & Taekwon, 2021). A positive attitude enhances students active performance which leads to success, while a negative attitude has a reflection of nonparticipation as an activity that leads to failure (Abim, 2009). A study conducted by Uwase and Edoho (2018) revealed that primary school students with positive attitudes toward learning mathematics performed well in mathematical tasks. Students with positive attitudes enjoy studying and practicing mathematics, which increases their competence (Aiken, 1970; Andamon & Tan, 2018; Ashcraft & Kirk, 2001). According to Mullis et al. (2020), students with a positive attitude toward mathematics were interested in participating in mathematics courses and spent more time studying mathematics than students with a negative attitude. Furthermore, the students with positive attitudes provided better performance in specific tasks in mathematics (Stephen & Evans, 2000; Uwase & Edoho, 2018). In contrast, students with a negative attitude toward mathematics perceived mathematics as an unnecessary subject and felt afraid to participate in courses dedicated to it (Guo et al., 2015; Wigfield et al., 2016). A study conducted by Jennison and Beswick (2010) on attitudes toward mathematics revealed that students with negative feelings about mathematics performed poorly, and they have inappropriate feelings about the subject. Consequently, students with positive attitudes toward mathematics perform much better than those with negative attitudes (Papanastasiou, 2000). Therefore, to succeed in mathematics, it is important to maintain a positive attitude toward it (Dowker et al., 2012). From this perspective, the relation between students' performance in mathematics and their attitude toward it is positive significantly (Andamon & Tan, 2018).

2.5. Contribution to The Literature and The Novelties

The effects of procedural teaching on students' performance in and anxiety about mathematics have been revealed by many previous studies (Ramirez et al., 2013). There are, however, fewer studies on the relationship between students' performance in mathematics and having conceptual knowledge (Price, 2015). For example, according to Zakaria et al. (2010), there is a positive association between conceptual knowledge and academic achievement in mathematics. In addition, some studies found the effect of conceptual teaching on students' anxiety in mathematics. For instance, according to Khoule et al. (2017) teaching conceptual mathematics decreases students' mathematics anxiety which leads to an increase in their performance. However, a study such as the present study that concentrates on how conceptual knowledge affects the three variables, students' achievement in, anxiety about, and attitude toward mathematics, could not be found in the scientific literature.

In the region where the current study took place, one preliminary study has been conducted on the topic by Hussein and Csíkos (2021) revealed that secondary school students in Erbil city had many problems with the concept of function. The study showed that students had difficulty defining the concept of function, and they could not recognize different representations of functions. However, a study that concentrates on how conceptual knowledge affects the students' anxiety or students attitudes toward mathematics could not be found in the area. Therefore, the present study will try to investigate, on one hand, how conceptual teaching impacts students' achievement in, anxiety about, and attitude toward mathematics in secondary school students in Erbil-Iraq. On the other hand, it tries to investigate the importance of conceptual knowledge in mathematics teaching from the perspectives of mathematics teachers in the same area.

3. Methodology

3.1. Participants

The sample study should be representative of the population of the study (Taherdoost, 2016). Individual semi-structured interviews were managed with secondary school mathematics teachers in the Kurdish Region of Iraq to investigate the research questions (Bryman, 2004). An online blog was made by the researcher that contained all details about the present study, namely, the title, research aim, the significance of the research, definitions, and call for voluntary participation. Subsequently, the blog link was sent to mathematics educators in Erbil, and they were contacted through phone, email, and social media. Consequently, the researcher recruited 30 secondary school mathematics teachers for the sample study. The researcher chooses the participants based on a set of basic criteria; they had to cover a range of geographical locations in the city and have different years of teaching experience. Most participants were male; in total, there were 11 women and 19 men. They have a bachelor certificate in mathematics. All of them had good enough teaching experience in mathematics, four participants had up to five years of teaching experience and the rest had over six years of teaching experience.

Furthermore, two hundred students in grade 8 participated in this study, 110 female and 90 male. They were 14 years old. The researcher used Purposive sampling to select three public secondary schools in Erbil, in the Kurdistan region of Iraq. According to the previous studies, on average, taking 200 students as a sample study is sufficient for this kind of study (see, for example, Andamon & Tan, 2018; Krejcie & Morgan,1970). In the schools in Erbil, each class contains approximately 30 to 35 students. Accordingly, six classes were taken, three of them were chosen randomly as experimental groups and the rest of them were control groups. In purposive sampling, candidates who have similar characteristics are considered and taken as a sample study (Etikan et al., 2016). The three schools out of a total 130 schools were chosen based on the similarity of some important characteristics: socioeconomic background, geographical location, and students' previous aptitude in mathematics and science. These three aspects were considered in sample selection to get an accurate result. In Erbil, people with different socioeconomic backgrounds live in different parts of geographical locations. For example, there is a Golden area where most of the people who live there have a rich economy. In these high socioeconomic status families, most of their children study in the top private schools, or their children have a private teacher for each subject. The aptitude of those students cannot equalize and combine with students who are from lower socioeconomic backgrounds. In the present study, the three schools were chosen in medium socioeconomic areas. Another aspect that was considered in choosing the sample study was, students' previous aptitude in mathematics and science, they were checked by looking at their last year's grades to equalize the groups. The three school administrators were asked to provide me the students' grades for mathematics and science subjects. The students who had on average less than 60% were sorted to a low level, an average of 60% to 80% sorted to a medium level, and an average of over 80% sorted to a high level. Then the groups were redistributed based on the students' level where each group containing an approximately equivalent amount of low, medium, and high-level students. To compare the three schools, the experimental and control groups were compared using a 3x2 ANOVA (school x group) with a pretest. The results show that the schools and classes do not differ in grades, anxiety, and attitudes at the beginning of the study.

In each of the three sample schools, there were two groups of students. The students at each school were divided into two numerically equivalent groups based on their average mathematics and science grades from the previous year. Consequently, the participants in all three schools represented two large groups whose average mathematics grades resembled one another. Each group in the two large groups contained low, average, and high achievers. In addition, they had the same ages, and they read the same mathematics subjects. One of the groups was designated the experimental group. The other was designated a control group; The groups were chosen randomly. Each group in each of the three schools was called a subgroup.

3.2.Instruments

This study aimed to collect both qualitative and quantitative data to investigate the answer to the four research questions. Therefore, three tools were used in the present study for data collection, interview, experiment, and questionnaire. The main reason behind selecting these three tools was the nature of the research questions (Thomas, 2009). The research tool is chosen based on what the researcher tries to discover (Cavaye, 1996). The interview was used to investigate the first research question that required expressing the mathematics educator’s idea. Experiment and questionnaire were utilized to investigate the last three research questions.

3.3.Research Focus and Design

Semi-structured interviews were conducted for this study. The interview questions were formulated according to three aims. The first was to reveal mathematics teachers’ familiarity with conceptual knowledge and teaching conceptually, in other words, their understanding of conceptual knowledge and the differences between conceptual and procedural teaching. The second was to reveal the importance of teaching mathematics conceptually from the perspectives of mathematics teachers, in other words, why teaching mathematics conceptually is important, and the teaching methods applied by teachers in the classroom. The final perspective was to identify obstacles that mathematics teachers face in teaching conceptual knowledge and how they can be managed.

In addition, the experimental approach was used to determine the impact of a manipulated variable: teaching for conceptual understanding (Sekaran, 1992). In the experimental (treatment) groups, mathematics instructors focused on teaching for conceptual understanding (see Appendix D). In the control groups, they taught mathematics subjects conventionally (Gürbüz et al., 2010). Conventional teaching in the Kurdistan region of Iraq mostly depends on communicating procedural knowledge. In procedural teaching, the teachers teach how to use the mathematics rules to solve the problem regardless of explaining the relationships between the concepts, and answer about how and why questions in the steps in the problem solving (see Appendix E). Teachers are prepared to teach both conceptual and procedural knowledge. Nonetheless, for many reasons, they focus on procedural understanding; mainly, it is more comfortable for them, the learning materials are available in textbooks, and it does not require them to change their teaching styles (Maryunis, 1989). Teachers in both groups had almost the same amount of teaching experience, which ranged from 8–11 years per person. The duration of the experiment was 5 weeks, beginning on May 16 and ending on June 20, 2021. In each group 25 lessons were taught; the period of each lesson was 40 minutes. In the experimental group's classes, teachers combined conceptual and procedural teaching by focusing on three main aspects: using instructional language carefully and avoiding naked numbers, focusing more on concepts than algorithms and shortcuts, and building connections among concepts. In addition, these teachers concentrated on providing in-depth explanations of the relationships among the steps required to solve a mathematics problem. This teaching method also utilized thinking aloud, one of the most common strategies in metacognition (Moghadam & Fard, 2011). Teaching mathematics conceptually is not a simple process; a certain kind of knowledge is required (Putnam et al., 1992). Nevertheless, I coordinated with mathematics teachers to successfully investigate the experimental process. I worked with mathematics teachers to design and deliver lessons that fostered conceptual knowledge of mathematics. Before each lesson in all groups, I had ten minutes meeting with the mathematics teachers who enrolment in the experimental program, to talk about the lesson and check that everything was well and going in the correct direction. Thereby, I attended most of the classes in both the experimental and control groups to observe the teaching processes and to ensure that the experiment proceeded correctly.

4. Data Analysis and Results

4.1. Interview

After the data collection stage, the researcher transcribed all audio recordings and read them several times for accuracy. For participants who did not consent to be recorded, the interviewer took notes during the interview. The interviews were transcribed on the same day that they took place (ideally directly after each interview) to reduce recall bias. In a process called condensation, the text was shortened while maintaining its core meaning (Erlingsson & Brysiewicz, 2017). To facilitate the analytical process, coding techniques were used to identify and record underlying ideas in the data. The coding process can be used to clarify, structure, and develop deeper meanings from the interview conversations. According to Erlingsson and Brysiewicz (2017, p. 2), “a code can be thought of as a label; a name that most exactly describes what this particular condensed meaning unit is about, usually one or two words long.” Deductive coding was used in the present study to focus on the research questions. Also known as concept-driven coding, the process of deductive coding begins with predefined codes, which are then applied to the new qualitative data (Medelyan, 2021). The next step was to categorize and group the codes to make sense of the data. Codes that are related in content or context can be grouped to make a category (Erlingsson & Brysiewicz, 2017). Next, three main themes were identified based on the aforementioned three main perspectives (Erlingsson & Brysiewicz, 2017). Then, different ideas and themes are related to each other to answer the research questions (Rubin & Rubin, 1995). See [Table 7].

Table 7: Data Analysis Classifications

Familiarity with conceptual knowledge	**Meaning of conceptual knowledge**	**Familiarity without any explanation:** Around 20% of participants (six interviewees) were familiar with the term “conceptual knowledge.”
		Familiarity with the researcher’s explanation: Seventy percent of participants (21 interviewees) recognized the meaning of conceptual knowledge after the researcher’s explanation.
	Differences between conceptual and procedural teaching	**Teaching conceptually, teaching procedurally:** Overall, 63.3% of participants (19 interviewees) named clear differences between the two teaching methods.
		Teaching conceptually is not important:

<table>
<tr><td rowspan="4">Perspectives on teaching mathematics conceptually</td><td rowspan="2">Perspectives on the importance of teaching mathematics conceptually</td><td>Overall, 6.6% of participants (two interviewees) believed that teaching mathematics conceptually was not important.</td></tr>
<tr><td>Teaching conceptually is important:
Overall, 93.3% of participants (28 interviewees) believed that teaching mathematics conceptually was important.</td></tr>
<tr><td rowspan="2">Teaching methods used by teachers in the classroom</td><td>Teaching procedurally:
Overall, 73.3% of participants (22 interviewees) only taught mathematics procedurally.</td></tr>
<tr><td>Teaching conceptually and procedurally:
Only 26.6% of participants (eight interviewees) combined conceptual and procedural teaching.</td></tr>
<tr><td colspan="2">Factors needed for conceptual teaching</td><td>- More time
- Training course for teachers
- Reducing the amount of students’ curriculum
- Teaching method in Kurdistan</td></tr>
<tr><td>Obstacles to teaching conceptual knowledge</td><td>Obstacles</td><td>- Insufficient time
- Insufficient knowledge among teachers.
- Pressure by school administrators and supervisors to complete the curriculum during the academic year.
- Some mathematics teachers believed that conceptual teaching complicates mathematics for students.
- Many students only want to pass their mathematics course rather than develop a deep understanding of the topic.</td></tr>
</table>

	Potential solutions	- Increase the duration and weekly frequency of mathematics classes. - Hold open training courses for mathematics teachers. - Encourage school administrators and supervisors to not only focus on completing the curriculum but also on achieving a better understanding. - Foster mathematics communities to exchange information. - Make contentment for mathematics teachers that teaching mathematics conceptually is not a waste of time and does not make mathematics more complicated for students, by academic debate.

4.2. Experiment and Survey

Different techniques were utilized to statistically analyze the data and thereby answer the last three research questions. Descriptive statistics—namely, standard deviation and mean—were used to describe students' performance in, anxiety about, and attitude toward mathematics (Andamon & Tan, 2018).

Descriptive statistics analysis shows that the mean score for Post-Grade in the experimental group increased more than in the control group compared to their Pre-Grade average. The mean score for the experimental group is 69.0 in Pre-Grade, but this mean score reached 72.1 in Post-Grade. However, in the control group, there is only a 0.5 difference between Pre-Grade and Post-Grade mean scores. The difference in mean scores between Pre-Attitude and Post-attitude in the treatment group is 14.5, which is much higher than the difference in the mean scores between Pre-Attitude and Post-Attitude in the control group 3.6. Meanwhile, in the experimental group, the student's anxiety was reduced according to Post-Anxiety test scores (mean = 26.5) compared to Pre-Anxiety (mean = 31.6). Nevertheless, in the control group, the mean score of students' anxiety slightly decreased from 31.8 to 30.6.

To determine whether the developmental sessions produced any improvement in mathematical achievement, grade scores were analyzed using mixed analyses of variance (2 groups × 2 genders × 2 measurements). The difference between the mathematical achievement pretest and posttest scores was significant: $F(1,196) = 18.48$, $p < .001$, partial eta squared = 0.086). Students achieved higher scores in the posttest than in the pretest. The difference between genders was not significant: $F(1,196) = 0.04$, $p = 0.83$, partial eta squared = 0.000. Likewise, the interaction between group and gender was not significant: $F(1,196) = 0.79$, $p = 0.375$, partial eta squared = 0.004. In contrast, the interaction of group × gender × measurement was significant: $F(1,196) = 4.52$, $p = 0.035$, partial eta squared = 0.023. Based on Tukey multiple comparisons, the experimental group's mathematical abilities improved significantly more for girls than boys. There was no change in the control group throughout the 5 weeks.

Mixed ANOVA measurements showed a statistically significant difference between the control group and the experimental group in terms of pretest and posttest scores: $p = 0.007$. The participants in the treatment group achieved a higher score than the participants in the control group in the achievement test.

According to the mixed ANOVA measurements, there was a statistically significant difference between the attitudes of the experimental and control groups: $F(1,198) = 149$, $p < 0.001$, partial eta squared = 0.429. The experimental group overperformed the control group in developing a positive attitude toward mathematics. Likewise, there was a statistically significant difference between the anxiety levels of the experimental and control groups: $F(1,198) = 117$, $p < 0,001$, partial eta squared = 0.372. This statistical analysis shows that the students' anxiety in the treatment group decreased more than the students' anxiety in the control group after the experiment period. In addition, it shows that the girls had higher anxiety than the boys: $F(1,196) = 4.33$, $p < 0.001$, partial eta squared = 0.022.

From that perspective, the three null hypotheses in the present study are rejected. The following substitutes replace them: "There is a statistically significant difference between the control group and the experimental group in terms of students' achievement," "There is a statistically significant difference between the control group and the experimental group in terms of the degree to which students' anxiety decreased," and "There is a statistically significant difference between the control group and the experimental group in terms of the degree to which students' positive attitude toward mathematics improved."

5. Discussion and Conclusion

5.1. Interview

The present study shows that the participants had some background in teaching conceptually. However, they did not try to teach conceptually in their classrooms. Only 20% of interviewees were able to define conceptual knowledge without any explanation from the researcher. When prompted, they remembered what they had learned about conceptual knowledge. This means that mathematics teachers in Kurdistan generally focus on teaching procedurally in their classrooms despite having some background in conceptual teaching.

Regarding teaching methods, only eight participants stated that they combined conceptual and procedural teaching. One of them said, “My students are very happy because I explain mathematics to them very clearly and deeply.” It seems that these students appreciate this teaching method. Another interviewee said, “It is true that I am a bit more tired than usual with teaching conceptually besides procedurally, but my students are comfortable because they can understand real mathematics.” The other participants used the procedural approach to mathematics teaching. Three participants believed that teaching mathematics in depth and explaining it in terms of relationships made mathematics too complicated for students; as a result, the students would dislike mathematics class even more. One interviewee said, “I explain mathematics rules to the students, and I teach them how to use those rules in solving mathematics problems. Why would I need to make the mathematics class more complicated by giving them deeper explanations?” Another stated, “We do not have problems with teaching procedurally, and my students’ grades are reasonable.” In addition, seven interviewees believed that teaching conceptually was only necessary for some subjects in mathematics. One said, “Some of the subjects in mathematics need conceptual teaching, but some others do not need it. For example, some very pure mathematics subjects can only be explained procedurally.” Six of the interviewees believed that teaching conceptually not only depended on teaching methods but also on the curriculum, school system, and school environment. One participant stated, “The four columns—the teacher, students, school system, and curriculum—are necessary to support the conceptual teaching of mathematics.”

In terms of solutions, interviewees mentioned that increasing the number of mathematics classes per week and class duration were needed to apply conceptual teaching. Interviewees believed that increasing the duration of mathematics classes from 40 minutes to 70 minutes and the number of mathematics classes from five to six classes per week should be considered. In addition, training sessions that focus on up-to-date teaching methods should be offered to mathematics teachers. One participant stated, "Mathematics teachers need to participate in training courses to develop their knowledge about teaching conceptual knowledge." Moreover, it is crucial for mathematics teachers to have their own communities in which to exchange knowledge and discuss teaching problems with mathematics experts. Furthermore, school administrators and supervisors should not only focus on completing the curriculum but also on teaching quality and ensuring students' understanding. Through academic debate, mathematics teachers must be persuaded that teaching mathematics conceptually is neither a waste of time nor makes mathematics more complicated for students. Finally, both students and teachers should be encouraged to focus on conceptual understanding alongside procedural understanding by formulating exam questions that require students to have conceptual knowledge to correctly answer.

Despite a noticeable shift in focus toward conceptual knowledge among researchers and educators, participants in this study mentioned many obstacles to teaching mathematics conceptually. First, conceptual knowledge can be implicit or explicit, which means that it might be not verbalizable (Goldin-Meadow et al., 1993). Only around half of the participants could differentiate between teaching mathematics conceptually and procedurally. Some of them confused the two teaching approaches, while others provided ambiguous answers (e.g., "By teaching conceptually, the mathematics teacher will connect the subject with our daily lives, while, by teaching procedurally, the mathematics teacher only focuses on pure mathematics."). Seven interviewees did not want to provide any explanation and said that they did not remember at the moment. The participants also believed that, for some advanced mathematics subjects, the topic cannot be explained in depth; instead, it can only be explained procedurally. This is consistent with previous studies that indicated that conceptual and procedural knowledge cannot be easily differentiated because they are so deeply intertwined (Baroody & Lai, 2007; Crooks & Alibali, 2014; Long, 2005; Star, 2005).

Another obstacle to conceptual teaching is pressure from supervisors and school administrators to complete the curriculum and increase students' pass rates. Thus, the focus is on quantity rather than quality in students' understanding of mathematics. According to Zakaria et al. (2010), school administrators encourage mathematics teachers to concentrate on student achievement in exams and on completing the curriculum regardless of students' satisfaction with mathematics courses or depth of understanding. Therefore, most mathematics assessments traditionally depend on students' ability to procedurally manipulate knowledge. Assessment tools focus on procedural knowledge rather than both conceptual and procedural knowledge (De Zeeuw et al., 2013).

Insufficient understanding of the nature and structure of mathematical knowledge is another reason why teachers focus on procedural knowledge rather than conceptual knowledge (Hallett et al., 2012; Lin et al., 2013). In this study, interviewees believed that mathematics teachers did not have enough knowledge to teach all mathematics subjects conceptually. Therefore, they proposed training courses for mathematics teachers and the development of communities for academic discussion with support from mathematics experts.

Procedural knowledge has become standard knowledge for solving mathematics problems. For example, students are graded on exams based on the number of correct answers (Rittle-Johnson & Siegler, 1998). This is consistent with the results of the present study, as some participants believed that teaching conceptually was not necessary because assessment tools are based on procedural knowledge. One of the participants said, "I tell my students, mathematics rules and how to use them for problem-solving. My students' grades are reasonable, and we do not have a problem with procedural knowledge."

Finally, some mathematics teachers believe that prioritizing conceptual knowledge is time-consuming compared to procedural knowledge because this requires more explanation and a deeper understanding of the topic (Baroody & Lai, 2007; Crooks & Alibali, 2014). Participants in this study confirmed that teaching conceptually is time-consuming, which is difficult to manage. For example, one interviewee said, "It is not easy to teach mathematics conceptually in the classroom within a 40-minute class period." However, according to Andrew (2019), it is time-consuming not to prioritize procedural knowledge over conceptual knowledge in mathematics teaching because students spend significant amounts of time not understanding what they are working on; as a result, mathematics courses become unpleasant and boring. Andrew provided two main reasons for this. Firstly, a better understanding of mathematics reduces the time that students spend being confused. If students do not understand key concepts, they struggle to remember rules and procedures. Secondly, students with a good understanding of mathematics require less practice. If mathematics teachers apply a conceptual approach to teaching first and a procedural approach later, students do not require much practice to solve problems (Andrew, 2019).

5.2. Experiment

This study shows that, in terms of mathematics achievement, there is a statistically significant difference between the experimental and control groups. The teachers in the treatment group followed lesson plans focused on conceptual understanding. Teachers in the control group followed the principles of conventional teaching. Teaching mathematics conceptually led students in the experimental group to achieve higher mathematics exam scores than students in the control group. The experimental group's mean achievement posttest score was much higher than its mean pretest score. In the control group, only a slight difference was recorded between the pretest and posttest scores. This finding indicates that conceptual teaching affected mathematics achievement in the experimental group. This finding is consistent with Khoule et al.'s (2017) study, which showed that the students in a conceptual group performed better on conceptual and procedural quizzes than those in a procedural group. Furthermore, students in the conceptual group were more able to reason logically, formulate solutions, and understand mathematics flexibly.

Teaching mathematics conceptually helped students achieve higher scores on mathematics exams because these students learned mathematics based on relations and concepts rather than procedures. Relational understanding helped the students remember mathematics rules more easily, and it provided them with the ability to adapt their knowledge to solve new mathematics problems. Students who possess relational understanding will be active in finding new areas in which to apply mathematics—for instance, they might apply it to the roots of trees that extend in all directions (Skemp, 1976). In contrast, it was difficult for students in the control group to apply what they learned in the classroom on the posttest. In particular, they failed to answer questions with different contexts from those that they solved during class. This finding is consistent with Zakaria et al.'s (2010) study, in which a significant relationship between conceptual knowledge and mathematics achievement was recorded. Conceptual teaching helps students to better achieve the learning process goal (Hurrell, 2021).

This study also found that teaching mathematics conceptually had a different impact on different genders. Female students in the experimental group achieved higher exam scores than male students. This result contradicts the findings of Hyde et al. (1990), who found no statistically significant difference in mathematics achievement between male and female students. This might suggest that female students have more mathematics anxiety than male ones (Beesdo et al., 2009). It was found that teaching for conceptual understanding reduces female students' anxiety more than it does male students'. Teaching for conceptual understanding reduces female students' anxiety more than it does male students'. As a result, female students achieved higher scores than male ones on the mathematics test. Students with lower levels of mathematics anxiety tend to have a stronger understanding of arithmetic than students with higher levels of mathematics anxiety (Price, 2015). However, girls have higher mathematical anxiety and this coupled with higher intrinsic motivation, can improve their math performance (Wang et al. 2015).

Students in the experimental group learned to self-monitor by asking themselves many questions during the problem-solving process. For instance, they asked, "How can I start to solve this problem?" "What is the relationship among these steps?" and "What is another method to solve this problem?". These kinds of questions help learners improve their metacognitive ability, which helps them to understand mathematics in a deeper and more meaningful way (Ilyas & Basir, 2019; Kramarski et al., 2002; Salehi, 2002). This finding is consistent with the findings of previous studies. For example, Amin and Sukestiyarno's (2015) study found that metacognitive knowledge plays an active role in improving students' achievement. According to Grant (2014), students' ability to monitor themselves during the learning process can increase their ability to solve problems.

Compared to students in the experimental group, students in the control group found it harder to remember mathematical rules during the posttest. Many students in the control group asked, "Teacher, could you explain to me which rule I have to use to answer question number *x*?" It seems that they had high levels of anxiety because teaching mathematics procedurally increased their anxiety rather than overcoming it (Khoule, et al., 2017). Moreover, high levels of anxiety make it difficult to remember mathematical rules (Rayner et al., 2009).

The present study statistically revealed that the conceptual method of teaching can overcome students' anxiety about mathematics. The traditional method, also known as the procedural method, cannot reduce students' anxiety because it depends on memorizing rules. The students had high levels of anxiety because they were taught mathematics procedurally which increased their anxiety rather than overcoming it (Khoule et al., 2017). Moreover, high levels of anxiety make it difficult for them to remember mathematical rules (Rayner et al., 2009).

For students to be successful in learning mathematics, teachers must take both cognitive and affective aspects into consideration (McLeod, 1994). In the present study, the effect of a conceptual teaching method on students' attitudes toward mathematics revealed that the attitudes of participants in the experimental group improved more than the attitudes of those in the control group. The experimental group's mean posttest score for positive attitude was higher than its mean pretest score, but this was not the case for the control group. According to Ashcraft and Kirk (2001), maintaining a positive attitude toward mathematics helps to increase students' competence in mathematics courses. This finding is consistent with a study conducted by Jennison and Beswick (2010) on the effect of an interactive teaching method on students' attitudes toward and performance in mathematics. The study's results revealed that the majority of participants increased their confidence and their positive attitudes toward mathematics by the end of the study. In addition, a study conducted by Zamir et al. (2022) on determining students' attitudes and achievements through problem-based learning, revealed that mathematics attitude is considered as a critical element in the process of mathematics learning. Confidence in learning mathematics and mathematics motivation had a significant role in the students' attitudes toward problem-based learning. Therefore, there is a strong association between students' attitudes toward mathematics and their performance in this subject. Students with a positive attitude toward mathematics have higher performance than whom have negative attitudes toward it (Naungayan, 2022; Segarra & Julià, 2021).

In summary, most of the interviewees in the present study believed that teaching both conceptual and procedural knowledge was necessary for students to better understand mathematics. this finding is consistent with Turner et al.'s (2002) study, which showed that using more than one teaching approach by the teachers helps the students to develop their confidence and achievement. However, the participants in the present study preferred procedural teaching for several reasons, the main ones being the pressure that they are under to complete the curriculum in an academic year and ensure a high pass rate among students and the fact that they regard teaching procedural knowledge as being easier.

Moreover, the results show that students will earn a higher score in mathematics if they participate in thinking and exploring rather than merely learning mathematics rules mechanically (Khoule et al., 2017). Teaching mathematics conceptually built confidence among this study's participants. It decreased their mathematics anxiety, and it helped them create the confidence needed to absorb new mathematical knowledge. To summarize, teaching mathematics conceptually not only improves students' achievement in mathematics but also reduces their mathematics anxiety and improves their positive attitudes toward this subject.

5.3. Conclusion

The study's main findings are detailed in this section, which also explains how they connect to the study's research questions and hypothesis. This last section provides an interpretation of the findings, brings the investigation to a close, and leaves the reader with a lasting impression of the study.

Mathematics does not consist of a collection of isolated facts and algorithms; rather, it is a web of interconnected elements (Nik Pa, 2003). Likewise, there is a relationship between conceptual knowledge and procedural knowledge; gains in conceptual knowledge lead to increases in procedural knowledge (Lauritzen, 2012; Rittle-Johnson et al., 2001; Rittle-Johnson et al., 2015). Therefore, this study investigates the importance of conceptual teaching in addition to procedural teaching in mathematics from the perspectives of mathematics teachers in Kurdistan, with the aim of disseminating the results. The study focuses on mathematics teachers' perspectives on teaching mathematics conceptually, the conditions needed to teach conceptually, and the obstacles that they face in teaching mathematics conceptually. Furthermore, it investigated how teaching mathematics conceptually impacts students' achievement in, anxiety about, and attitudes toward mathematics.

The results revealed that most interviewees believed that teaching conceptual knowledge alongside procedural knowledge was crucial for students to have a better understanding of mathematics (Nahdi & Jatisunda, 2020). This finding is consistent with previous studies. For example, the National Governors Association Center for Best Practices and Council of Chief State School Officers (2010) stated that, by focusing on conceptual knowledge in mathematics teaching, students would gain a deeper understanding of mathematics and that information would be retained for a longer period of time. To improve learning quality and student achievement, it is vital to help students to understand mathematics conceptually. Once students gain conceptual knowledge, they can assess the suitable procedure to use in a specific mathematical problem (Brownell, 1945; Schneider & Stern, 2010).

However, only a few interviewees in the present study combined procedural and conceptual teaching in their classes. Participants saw teaching conceptually as time-consuming because it requires more explanation and a deeper understanding of the topic. Others believe that teaching conceptually was not necessary because it made mathematics more complicated for students. These reasons discouraged mathematics teachers from teaching conceptually.

Moreover, in this study experimental approach was utilized. The students in both groups (experimental and control group) have similar experiences and the same academic background. The students in both groups are provided the same mathematics activities during the five weeks of the experiment. The only difference between them was the teaching method. In the experimental group, conceptual teaching was focused on, while the control group was taught traditionally. The teaching process in both groups was observed during the experiment by the researcher.

The results from the experiment revealed that there are statistically significant relationships between teaching for conceptual understanding and the three aforementioned variables. Compared to the control group, the experimental group performed better in a mathematics exam. This result indicates that participants in the experimental group outperformed the control group in mathematics achievement, reduced their mathematics anxiety, and improved their attitudes toward mathematics.

5.4. Recommendations and Suggestions

Based on the results of the present study, some recommendations were formulated for mathematics teachers, school administrators, supervisors, education directors, and the Ministry of Education, Kurdistan of Iraq.

The conventional teaching method should be revised to match the skills that students need to be productive (Rossnan, 2006). Students need practical mathematics classes, and they should be involved in thinking and analyzing rather than merely learning the rules and applying them (Curtain-Phillips, 1999). When learning mathematics, metacognition must be emphasized as much as cognition because it is one of the main elements in students’ achievement. Therefore, it is recommended that mathematics teaching focuses on both conceptual knowledge and procedural knowledge since conceptual teaching is key to a better understanding of mathematics among students.

Through academic debate, education directors, school administrators, and supervisors should persuade mathematics teachers that teaching mathematics conceptually is not a waste of time and does not make mathematics more complicated for students.

Increasing the frequency of mathematics classes from five to six classes per week is recommended to allow teachers to have more time to explain mathematics. And exam questions should be formulated in a way that requires students to have a conceptual understanding of the topic; this would motivate both students and teachers to focus on conceptual understanding alongside procedural understanding.

Educators should encourage students to acquire the ability to confront mathematics problems (Furner & Berman, 2003). Students are often afraid of mathematics, which generates anxiety more than other disciplines (Shore, 2005). According to one study, almost two-thirds of adults in the United States had a deep fear of mathematics (Burns, 1998).

However, anxiety can be controlled and reduced (Tobias, 1993). To reduce mathematics anxiety, educators must focus less on speed tests (Reys et al., 1995). Instead, they have to focus more on adopting new teaching methods that are conformable to mathematics (Zamir et al., 2022). Additionally, mathematics teachers must make mathematics an enjoyable subject by using a meaningful method of teaching, and they must also explain the importance of mathematics to everyday life and to students' future careers (Cruikshank & Sheffield, 1992). Furthermore, Woolfolk (1995), provided mathematics teachers some points to avoid students from mathematics anxiety, mainly, all instructions should be clear for students, avoid the pressure of tests by deleting unnecessary pressure parts, and promote the student positive behavior.

I have the same belief as Taylor and Brooks (1986): both basic concepts and correct procedures are important when solving mathematics problems. Therefore, I recommend that mathematics teachers should use multiple teaching methods to build connections between abstract thought and conceptual learning (Hurrell, 2021).

5.5. Limitation of the Study

To provide a more precise and insightful interpretation of the findings, researchers must be aware of and open about the limitations of their study. Researchers can better comprehend their research's advantages and disadvantages, as well as identify areas that may need further investigation or development, by being open and honest about the study's limits. The limitation of the study is characteristics that affect the generalization and interpretation of the result of the study (Price & Murnan, 2004).

This study like the previous studies has some limitations. One limitation of this study is that the study focused only on students in secondary schools in the center of the city, of Erbil. Therefore, I do not know what would be the result if I use schools in villages for this research. Another limitation of this study is I do not use other variables that are related to mathematics, for example, special ability and memory. Finally, there is a question so far whether there is a difference between 13-14 years old students who are at the beginning of the formal operational stage (Piaget's formal operational stages) and 16-17 years old students (Kuhn, 1979).